XIV. " On the Enumeration of x-edra having an $(x-1)$-gonal
Face, and all their Summits Triedral." By the Rev. THO-
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The object of the paper is to enumerate the x-edra which have an
$(x-1)$-gonal face, and all their summits triedral; or, what is the
same thing, to find the number of the x-acra which have an $(x-1)$-
edral summit, and all their faces triangular.

Every x-edron having an $(x-1)$-gonal face has at least two trian-
gular faces. Let A be an x-edron having all its summits triedral,
and having about its $(x-1)$-gonal face k triangular faces. Suppose
all these triangles to become infinitely small; there arises an $(x-k)$-
edron B, having an $(x-k-1)$-gonal face, and all its summits tri-
edral. B will have k' triangular faces, k' being not less than two,
nor greater than k. And there is no other $(x-k)$-edron but B,
which can arise from the vanishing of all the k triangles of A ; *i. e.*
there is no $(x-k)$-edron but B, from which A can be cut by re-
moving k of the summits of B in such a way as to leave none of its
k' triangles untouched.

If we next suppose the k' triangles of B to vanish, there will arise
an $(x-k-k')$-edron C, having an $(x-k-k'-1)$-gonal face, all its
summits triedral, and k'' triangular faces, k'' not <2, nor $>k'$. And
thus we shall at last reduce our x-edron, either to a tetraedron, or
to a pentaedron having triedral summits.

All x-edra here considered fall into six varieties, differing in the
sequence of the $x-1$ faces that are collateral with the $(x-1)$-gonal
base. They are either *irreversible*, as the octaedron 6435443, the
seven faces about the base reading differently both backwards and
forwards from every face ; or *doubly irreversible*, as the heptaedron
543543, whose six faces about the base are a repetition of an irre-
versible period of three ; or *triply irreversible*, as the decaedron
643643643, whose faces exhibit a thrice-repeated irreversible period;
or they are *reversible*, *doubly reversible*, or *triply reversible*, as the
hexaedron 53443, the enneaedron 63536353, or the heptaedron
535353, exhibiting a single, double, or triple period, all reading

backwards and forwards the same. If P_x be the number of x-edra having an $(x-1)$-gonal base, and all their summits triedral,

$$P_x = I_x + I_x^2 + I_x^3 + R_x + R_x^2 + R_x^3,$$

the symbols on the right denoting the numbers of x-edra of the six varieties that make up P_x.

Each variety is again subdivided according to the number of triangular faces. Thus, if $P(x, k)$ denote the number of x-edra on an $(x-1)$-gonal base, having k triangular faces, and all their summits triedral,

$$P(x, k) = I(x, k) + I^2(x, k) + I^3(x, k) + R(x, k) + R^2(x, k) + R^3(x, k).$$

The number k is not <2, nor $>\dfrac{x-1}{2}$, and $P_x = \Sigma P(x, k)$, for all values of k.

It is necessary to solve the following

Problem.—To determine the number of $(x+k+l)$-edra, none of which shall be the reflected image of another, that can be made from any x-edron having k triangular faces, by removing $k+l$ of its base-summits, thus adding $k+l$ triangular faces, so that none of its k triangular faces shall remain uncut.

The x-edron is supposed to have an $(x-1)$-gonal face, and all its summits triedral; no edge is to be removed, and $k+l$ not $>x-1$.

When the x-edron, the subject of operation, is *irreversible*, all the resulting $(x+k+l)$-edra will be irreversible. If it is *reversible*, some of them will be reversible and others irreversible; if it is *multiple*, some of them will be, and others will not be, multiple.

If the subject of operation is irreversible, the number required by the problem is

$$ii(x,k,l) = 2^k \cdot \frac{x-1-k^{l|-1}}{\underline{|l+1}} - \Sigma_a(2^a - 1) \cdot 2^{k-a} \frac{k^{a|-1}}{\overline{|a+1}} \cdot \frac{x-1-2k^{l-a|-1}}{\overline{|l-a+1}},$$

taken for all values of a not greater than the least of k and l; i. e. $k-a$ not <0, 0 not $>l-a$.

The complete answer to the problem is expressed by the following equations, in which, of the capitals on the left, the first expresses the result, and the second the subject of operation. That is, $IR^2(x, k, l)$ denotes the number of irreversible $(x+k+l)$-edra having $k+l$ triangular faces about the $(x+k+l-1)$-gonal base, that can

be cut from any doubly reversible x-edron having k triangles about its $(x-1)$-gonal base.

Whenever k or l in the function $ii(x, k, l)$ is not integer, the function, by a geometrical necessity, is to be considered $=0$.

$\mathrm{II}(x, k, l) = ii(x, k, l)$,

$\mathrm{II}^2(2x+1, 2k, l) = \frac{1}{2}\{ii(2x+1, 2k, l) - ii(x+1, k, \frac{1}{2}l)\}$,

$\mathrm{II}^3(3x+1, 3k, l) = \frac{1}{3}\{ii(3x+1, 3k, l) - ii(x+1, k, \frac{1}{3}l)\}$,

$\mathrm{I}^2\mathrm{I}^2(2x+1, 2k, l) = ii(x+1, k, \frac{1}{2}l)$,

$\mathrm{I}^3\mathrm{I}^3(3x+1, 3k, l) = ii(x+1, k, \frac{1}{3}l)$;

$\mathrm{RR}(2x+1, 2k, l) = ii(x+1, k, \frac{1}{2}l)$,

$\mathrm{RR}(2x+1, 2k+1, l) = ii(x, k, \frac{1}{2}(l-2))$,

$\mathrm{RR}(2x, 2k, l) = ii(x, k, \frac{1}{2}l) + ii(x, k, \frac{1}{2}(l-1))$;

$\mathrm{IR}(2x+1, 2k, l) = \frac{1}{2}\{ii(2x+1, 2k, l) - ii(x+1, k, \frac{1}{2}l)\}$,

$\mathrm{IR}(2x+1, 2k+1, l) = \frac{1}{2}\{ii(2x+1, 2k+1, l) - ii(x, k, \frac{1}{2}(l-2))\}$,

$\mathrm{IR}(2x, 2k, l) = \frac{1}{2}\{ii(2x, 2k, l) - ii(x, k, \frac{1}{2}l) - ii(x, k, \frac{1}{2}(l-1))\}$;

$\mathrm{R}^2\mathrm{R}^2(4x+1, 4k, l) = ii(x+1, k, \frac{1}{4}l)$,

$\mathrm{I}^2\mathrm{R}^2(4x+1, 4k, l) = \frac{1}{2}\{ii(2x+1, 2k, \frac{1}{2}l) - ii(x+1, k, \frac{1}{4}l)\}$,

$\mathrm{RR}^2(4x+1, 4k, l) = ii(2x+1, 2k, \frac{1}{2}l) - ii(x+1, k, \frac{1}{4}l)$,

$\mathrm{IR}^2(4x+1, 4k, l) = \frac{1}{4}[ii(4x+1, 4k, l) + 2ii(x+1, k, \frac{1}{4}l)$
$\qquad - 3ii(2x+1, 2k, \frac{1}{2}l)]$;

$\mathrm{R}^3\mathrm{R}^3(6x+1, 6k, l) = ii(x+1, k, \frac{1}{6}l)$, $\mathrm{R}^3\mathrm{R}^3(7, 3, 3) = 1$,

$\mathrm{I}^3\mathrm{R}^3(6x+1, 6k, l) = \frac{1}{2}\{ii(2x+1, 2k, \frac{1}{3}l) - ii(x+1, k, \frac{1}{6}l)\}$,

$\mathrm{RR}^3(6x+1, 6k, l) = ii(3x+1, 3k, \frac{1}{2}l) - ii(x+1, k, \frac{1}{6}l)$, $\mathrm{RR}^3(7, 3, 1) = 2$,

$\mathrm{IR}^3(6x+1, 6k, l) = \frac{1}{6}\{ii(6x+1, 6k, l) + 3ii(x+1, k, \frac{1}{6}l)$
$\qquad - ii(2x+1, 2k, \frac{1}{3}l) - 3ii(3x+1, 3k, \frac{1}{2}l)\}$,

$\mathrm{IR}^3(7, 3, 2) = \mathrm{IR}^3(7, 3, 1) = \mathrm{IR}^3(7, 3, 0) = 1$;

$\mathrm{I}^n\mathrm{R}^m(x+1, k, x-k) = 0$.

By the aid of the above, together with the following, equations, the $(x+k+l)$-edra having $k+l$ triangular faces, an $(x+k+l-1)$-gonal base and triedral summits, are successively found.

$\mathrm{I}(x+k+l, k+l) = \Sigma\{\mathrm{I}(x+k') \cdot \mathrm{II}(x, k', l') + \mathrm{I}^2(x, k') \cdot \mathrm{II}^2(x, k', l')$
$\qquad + \mathrm{I}^3(x, k') \cdot \mathrm{II}^3(x, k', l') + \mathrm{R}(x, k') \cdot \mathrm{IR}(x, k', l')$
$\qquad + \mathrm{R}^2(x, k') \cdot \mathrm{IR}^2(x, k', l') + \mathrm{R}^3(x, k') \cdot \mathrm{IR}^3(x, k', l')\}$; &c.&c.

taken for all values of $k' + l' = k + l$.

Similar equations are to be formed for the remaining five sub-divisions of $P(x+k+l, k+l)$.

Of the products under Σ, the first factors are found by the preceding part of the process, and the second are given by the equations above written as solutions of the problem. The factors will of course frequently be zeros. Finally, if $x'=x+k+l$,

$$P_{x+k+l}=P_{x'}=P(x', 2)+P(x', 3)+\ldots+P(x', \tfrac{1}{2}(x'-1)).$$

Thus, to give an example,

$$P_{11}=P(11,2)+P(11,3)+P(11,4)+P(11,5)$$
$$=I(11,2)+I(11,3)+I(11,4)+(I(11,5)=0)$$
$$+I^2(11,2)+I^2(11,4)$$
$$+R(11,2)+R(11,3)+R(11,4)+R(11,5).$$
$$I(11,2)=I^2(9,2).II^2(9,2,0)+I(9,2).Ii(9,2,0);$$
$$I(11,3)=I(8,2).II(8,2,1)+R(8,2).IR(8,2,1)+I(8,3).I(8,3,0);$$
$$I(11,4)=I^2(7,2).II^2(7,2,2)+R(7,2).IR(7,2,2)$$
$$\qquad +R^3(7,3).IR^3(7,3,1);$$
$$I^2(11,2)=I^2(9,2).I^2I^2(9,2,0);$$
$$I^2(11,4)=I^2(7,2).I^2I^2(7,2,2);$$
$$R(11,2)=R(9,2).RR(9,2,0);$$
$$R(11,3)=R(8,2).RR(8,2,1);$$
$$R(11,4)=R(7,2).R(7,2,2)+R^3(7,3).RR^3(7,3,1);$$
$$R(11,5)=R(6,2).RR(6,2,3).$$

The result is

$$P_{11}=I_{11}+I_{11}^2+R_{11}=61+7+12=80.$$

XV. "Notes on British Foraminifera." By J. GWYN JEF-
FREYS, Esq., F.R.S. Received June 19, 1855.

Having, during a great many years, directed my attention to the recent Foraminifera which inhabit our own shores, I venture to offer a few observations on this curious group, as Dr. Carpenter, who has favoured the Society with an interesting and valuable memoir on the subject, seems not to have had many opportunities of studying the animals in the recent state.